Central Bank Exit Strategies and Credibility:

Some Implications from its Dynamic Optimizing Behavior

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Abstract

In this paper, we examine a problem of credibility that a central bank might face when exiting unconventional monetary policy. We develop a simple dynamic optimization model of a central bank, in which the bank's profit affects its balance sheet. The model derives the transversality condition that is necessary for the central bank to be sustainable and to conduct an optimal monetary policy after exiting monetary easing. In this sense, the transversality condition needs to be satisfied to maintain central bank credibility. We discuss some factors affecting the transversality condition and show that the central bank's balance sheet should be sound enough to generate no sustained loss.

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1. Introduction

In this paper, we examine a problem of credibility that a central bank might face when exiting unconventional monetary policy. We develop a simple dynamic optimization model of a central bank, in which the bank's profit affects its balance sheet, and we show that the bank's unsound balance sheet might create difficulty in conducting appropriate monetary policy and thus jeopardize its credibility.

Since the "Lehman shock" in 2008, many central banks in the industrialized countries have been using the unconventional measures of monetary easing policy. One of the main measures is quantitative easing, where a central bank expands the quantity of fund supply beyond the zero interest rate point. Another one is credit easing, where a central bank purchases some risky assets.

Many have studied whether such unconventional monetary easing measures are effective or not, and it still remains controversial. Whether they are effective or not, it is better to use such measures so long as they have no possibility of creating any problem. However, some argue that they jeopardize central bank credibility, especially when exiting the monetary easing. A central bank needs credibility that it is sustainable and maintains its ability to perform its functions such as stabilizing prices.[1] If the unconventional measures damage a central bank's balance sheet, then the central bank loses such credibility. Therefore, we need to examine the effectiveness of unconventional measures on one hand, and to assess the cost or risk of damaging the credibility on the other.

Though it is an important issue, surprisingly few studies, as compared

[1] Note that the credibility here is not the one that a central bank keeps a promise to fight against inflation when it has the ability to do so, as Barro and Gordon [1983] has examined.

with those on policy effectiveness, have analyzed central bank credibility in this context. Among few are Stella [1997, 2003], Bindseil *et al*. [2004], Ize [2005], Klüh and Stella [2008], Cincibuch *et al*. [2009], Adler *et al*. [2012] and Tanaka [2013], and they contend that an unsound balance sheet and low/negative profit of central bank create difficulty in conducting appropriate future monetary policy, which leads to jeopardizing central bank credibility.

These precedent studies have discussed the influence of unsound balance sheet on future monetary policy without showing any model. Bindseil *et al*. [2004], Ize [2005], and Cincibuch *et al*. [2009] are exceptions, but their models are not derived from any optimizing behavior of a central bank.[2]

In this paper, we develop a formal central bank model from its dynamic optimizing behavior. Our model incorporates the central bank's balance sheet and profit on infinite time horizon and shows their influence on monetary policy. From this model, we derive the condition that a central bank must satisfy in order to conduct appropriate monetary policy. If the bank's balance sheet is unsound, the condition is not satisfied, and the bank has difficulty in conducting appropriate monetary policy and thus loses central bank credibility.

This paper is organized as follows. In Section 2, we discuss what is considered to jeopardize central bank credibility, and we argue that its sound balance sheet is important. Then, we develop a dynamic optimization model of a central bank in Section 3, and analyze the balance

[2] Berriel and Bhattarai [2009] is probably the only study to set up a model of central bank optimizing behavior with its balance sheet and profit constraints. They do not, however, derive any explicit optimal solution but run a simulation instead. With the simulation, they examine not central bank credibility but the monetary policy where a central bank targets its own capital together with inflation and output gap.

sheet, monetary policy, and credibility in Section 4. Section 5 summarizes the analysis in this paper.

2. Central Bank Credibility

2.1. Capital, Profit, and Credibility

Bank credibility depends on the possibility that a bank is sustainable and will not fail. Capital is considered to be important for any bank, but private banks do not fail immediately when capital becomes negative. With negative capital, sooner or later they face the difficulty in raising necessary liquidity, and liquidity shortage leads them to bankruptcy. Like private banks, central banks do not fail with negative capital. Unlike private banks, central banks do not fall short of liquidity, because they can create liquidity for themselves.

However, this does not necessarily mean that central banks do not need any capital to maintain credibility. A central bank with less capital tends to generate less profit. If it continues to make losses, it continues to create liquidity to finance them. Since supplying more liquidity means monetary easing policy, it puts an obstacle to conducting monetary tightening policy when necessary. This possibly leads inflation to be out of control and, in this sense, it might jeopardize central bank credibility.

Thus, the important factor for credibility is whether or not a central bank does not make sustained losses. It depends not only on capital but also on other financial elements that affect present and future profits. Central banks need their balance sheets sound enough to generate no sustained loss even when they suffer shocks from large financial fluctuations, domestic or overseas.

Ueda [2004] and Ize [2005] have examined several cases of troubled central banks. For example, the central banks in Venezuela, Jamaica, and

Costa Rica had negative capital due to foreign exchange losses or the cost to deal with domestic financial crisis. This caused the rise in the interest rate on borrowings by central banks and expanded the losses. The banks had difficulty in stopping monetary easing policy due to the expansion in interest payment, which accelerated inflation.

2.2. The Case of the Bank of Japan

For more than a decade, the Bank of Japan (BOJ) has been taking aggressive monetary easing policy, including some unconventional measures. While effectiveness of such unconventional measures is a controversial issue, some are concerned about the risk that such measures might damage the BOJ's credibility in the future. In this subsection, we discuss the recent situation of the BOJ and see what might be a problem.

The BOJ has taken unconventional measures in three periods: the periods of zero interest rate policy (February 1999 to August 2000), quantitative easing (March 2001 to March 2006), and the policy after Lehman shock (from September 2008 to the present). The BOJ is now strengthening such measures by starting "new phase of monetary easing" in April 2013. In all of these periods, the policy rate was set nearly at zero, and the BOJ kept supplying more funds to the private sector.

Figure 1 shows that the monetary base was expanded drastically in the above three periods. Not only in the quantitative easing period, but also in the other two periods, the BOJ was taking the quantitative easing measure. The main measure of the monetary base expansion was outright purchases of Japanese Government Bonds (JGBs) as the figure shows. The BOJ also took credit easing by purchasing some risky assets, especially in the past few years, and total holdings of such risky assets amount to 9.6 trillion yen as shown in Table 1.

Insert Figure 1 and Table 1 around here.

There are two main concerns about the BOJ's credibility. One is the risk with risky assets and foreign assets. Fortunately, the holdings of these assets are not large up to now, but they are increasing. The other is the risk with the huge holdings of JGBs. Their price fluctuates, and the BOJ bears a large capital loss if the price falls. A fall in price can be caused by Japanese government's loss in credibility or a rise in the interest rate when the economy is exiting a slump. The BOJ experienced the latter, so we examine its experience in the next subsection.

2.3. Exit Strategies

It is an important issue how to absorb a large amount of liquidity at the exit of quantitative easing, which is called exit strategy. The BOJ accomplished such absorption when it ended the quantitative easing in March 2006.

The BOJ's exit strategy can be examined by Table 2. The table shows the changes in the balance sheet components corresponding to the decrease in monetary base during the period of half a year and the period of two years and a half after ending the quantitative easing. The BOJ decreased the monetary base by more than 20 trillion yen within half a year. Though it used mainly JGB purchases to expand the monetary base as seen in Figure 1, it used two measures to shrink it. In the first half a year, it used the measure of the short-term operations such as funds-supplying operations against pooled collateral, RAs, and bills. It should be noted that the BOJ decreased the funds-supplying operations, not increasing the funds-absorbing operations. Two years and a half after, it used the other measure, which is a decrease in the JGB holdings. It should be noted that it never sold any JGBs but waited them to be redeemed.

Insert Table 2 around here.

At the exit of quantitative easing, the interest rates were rising and asset prices including that of JGBs were falling. To avoid any capital loss, the BOJ did not sell any JGBs. It waited till they were redeemed, and for the meantime it absorbed the liquidity by shrinking the funds-supplying operations.

The BOJ succeeded in exit strategy without bearing any capital loss, but such an exit strategy is not always feasible. Since most JGBs are long-term bonds, the BOJ seemed to have prepared for the exit by purchasing those with shorter maturity period in order to have many of them redeemed within a couple of years. It decreased the funds-supplying operations, but to decrease them it needed to have them expanded before the exit. Hence, it seemed that the BOJ prepared for the exit carefully.

However, we cannot expect that a central bank can always prepare for the exit beforehand. The exit may not be predictable, or a sudden external shock, such as an oil shock, may hit the economy so that the central bank needs to absorb liquidity immediately. In these cases, such an exit strategy as the one by the BOJ in 2006 is not possible.

Bernanke [2009] proposes two measures in these cases. One is using reverse repos, and the other is to pay high interest on private banks' balances at a central bank to have the balances increased. Both measures are funds-absorbing measures. They decrease the monetary base by expanding the central bank liabilities, not by shrinking the central bank assets as the BOJ did. Since the interest rates should rise after the exit, large expansion in interest bearing liabilities might impose losses on the central bank, which might damage its balance sheet and credibility.

Thus, it should be emphasized that the funds-absorbing strategy is different from the funds-supplying strategy. Though both change the monetary base, the former strategy changes the size of other liabilities on a central bank's balance sheet, while the latter strategy changes the size of assets. The BOJ was successful by using the latter, and Bernanke proposes the former when the latter strategy is not available. Our model in the next section takes into account the difference between funds-supplying and funds-absorbing strategies. With the model, we examine the relation of the balance sheet and profit with monetary policy and credibility, and we show that the relation differs depending on the funds-supplying or funds-absorbing strategies.

3. A Simple Dynamic Optimization Model of a Central Bank

3.1. Model Setting

In this section, we develop a simple dynamic optimization model of a central bank. We incorporate the central bank's balance sheet and profit on infinite time horizon into the central bank behavior. With the model, we examine the bank's behavior at an exit of monetary easing.

At an exit, a central bank needs to stop the expansion of monetary base, and so it is assumed to minimize the following quadratic loss function:

$$\min_{\Delta H_t} \quad L = \sum_{t=1}^{\infty} \beta^t \left(\frac{1}{2} \Delta H_t^2 \right). \tag{1}$$

ΔH_t is a change in monetary base at t, and β is a discount factor. The bank has been taking a quantitative easing by keeping positive ΔH_t up to t=0, and it starts an exit strategy to reduce ΔH_t at t=1,.., ∞.[3] With uncertainty, we need an expectation operator, but we assume certainty for simplicity, since introducing uncertainty does not change any important implications discussed in this paper.

If it can set ΔH_t freely, as usually presupposed in many precedent studies, the central bank sets ΔH_t=0 for t=1,.., ∞. It may not be the case, however, if we take into account the central bank's balance sheet and profit, which impose constraint on its behavior.

The central bank's balance sheet in our model is Table 3, where A_t is the assets with interest rate r_{At}, H_t is the monetary base, B_t is the other liabilities that are all assumed to bear interest at the rate r_{Bt}, and K_t is the capital. Its profit π_t is,

$$\pi_t = r_{At}A_t - r_{Bt}B_t - O, \qquad (2)$$

where O is the central bank operation expenditures, and the profit is added to the capital in the next period:

$$K_t = K_{t-1} + \pi_{t-1}. \qquad (3)$$

K_t and ΔH_t can be negative, while A_t and B_t should be non-negative.

Insert Table 3 around here.

At the beginning of t=1, the central bank sets ΔH_t for t=1, …, ∞ given the initial conditions and exogenous variables. The initial conditions are A_0, B_0, H_0, K_0, and π_0 that is determined by the other initial variables. The exogenous variables are r_{At}, r_{Bt}, and O. To exit monetary easing and

[3] The exit strategy in our model is only to reduce the expansion of monetary base, while the BOJ took more aggressive one in 2006; it shrank the quantity of monetary base. Such an aggressive strategy can be examined by modifying the objective loss function, and it remains for the future study.

reduce ΔH_t, the central bank can take the strategy to change either B_t or A_t, and we discuss each strategy in the subsequent subsections.

3.2. The Model with Changes in Liabilities

In this subsection, we discuss the strategy where the central bank changes the interest bearing liabilities B_t to reduce the monetary base growth. As discussed in the Subsection 2.3, the measures to change the assets are not always available, especially when exit strategy requires quick monetary tightening policy. If not available, the central bank needs to expand the liabilities such as reverse repos for tightening policy.

We assume $A_t = \overline{A}$. With this and equation (3), the following balance sheet constraint applies:

$$H_t + B_t = H_{t-1} + B_{t-1} - \pi_{t-1}.$$

Using equation (2), this constraint becomes,

$$B_t + \Delta H_t = (1 + r_{Bt-1})B_{t-1} - r_{At-1}\overline{A} + O; \qquad \Delta H_t = H_t - H_{t-1}. \tag{4}$$

We also need another constraint, $B_t \geq 0$, but we neglect it to simplify the model handling and restrict our discussion to the case of non-negative B_t.[4] Thus, the central bank minimizes the loss function (1) subject to (4) with respect to ΔH_t and B_t. ΔH_t is a control variable, and B_t is a state variable.

We set the Lagrangian V, where λ_t is the Lagrangian multiplier.

$$V = \sum_{t=1}^{\infty} \beta^t \left[\frac{1}{2} \Delta H_t^2 + \lambda_t \left\{ (1 + r_{Bt-1})B_{t-1} - r_{At-1}\overline{A} + O - B_t - \Delta H_t \right\} \right]. \tag{5}$$

The first order conditions are as follows.[5]

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[4] We can modify our model to have the non-negative constraint on B_t by using Kuhn-Tucker theorem. In that case, when the non-negative constraint is binding, ΔH_t cannot be controlled by the central bank but determined by equation (4).

[5] To check the second order condition, it is easier to calculate by substituting (4) into (1) to eliminate ΔH_t:

$$\partial V / \partial \Delta H_t = \beta^t (\Delta H_t - \lambda_t) = 0, \tag{6a}$$

$$\partial V / \partial B_t = -\beta^t \lambda_t + \beta^{t+1} \lambda_{t+1} (1 + r_{Bt}) = 0, \tag{6b}$$

$$\partial V / \partial \lambda_t = \beta^t \left[(1 + r_{Bt-1}) B_{t-1} - r_{At-1} \overline{A} + O - B_t - \Delta H_t \right] = 0. \tag{6c}$$

From equations (6a) and (6b),

$$\beta (1 + r_{Bt}) \Delta H_{t+1} = \Delta H_t. \tag{7}$$

The following transversality condition must be satisfied:

$$\lim_{T \to \infty} \beta^T \lambda_T B_T = 0. \tag{8}$$

Equation (8) can be rewritten,

$$\lim_{T \to \infty} \beta^T \lambda_T B_T = \lim_{T \to \infty} \beta^T \lambda_T \prod_{t=1}^{T} (1 + r_{Bt-1}) \frac{B_T}{\prod_{t=1}^{T} (1 + r_{Bt-1})}.$$

From equation (6b), $\beta^T \lambda_T \prod_{t=1}^{T} (1 + r_{Bt-1})$ is constant at any T, so the transversality condition reduces to,

$$\lim_{T \to \infty} \frac{B_T}{\prod_{t=1}^{T} (1 + r_{Bt-1})} = 0. \tag{8'}$$

This condition implies that B_T should not grow faster than its interest payment.

3.3. The Model with Changes in Assets

The other strategy is that the central bank changes the asset holdings A_t to control the monetary base. We assume $B_t = \overline{B}$, and the balance sheet

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$$\min_{B_t} \quad L = \sum_{t=1}^{\infty} \beta^t \left[\frac{1}{2} \left\{ (1 + r_{Bt-1}) B_{t-1} - r_{At-1} \overline{A} + O - B_t \right\}^2 \right].$$

The second order condition is always satisfied as follows:

$$d^2 L / dB_t^2 = \beta^t + \beta^{t+1} (1 + r_{Bt})^2 > 0.$$

constraint is,

$$A_t = A_{t-1} + \pi_{t-1} + \Delta H_t .$$

Using equation (2), this constraint becomes,

$$A_t - \Delta H_t = (1 + r_{At-1})A_{t-1} - r_{Bt-1}\overline{B} - O . \tag{9}$$

As in Subsection 3.2, we neglect the non-negative constraint and discuss only the case of $A_t \geq 0$. The central bank minimizes the loss function (1) subject to (9) with respect to ΔH_t and A_t.

We set the Lagrangian V:

$$V = \sum_{t=1}^{\infty} \beta^t \left[\frac{1}{2}\Delta H_t^2 + \lambda_t \left\{ (1 + r_{At-1})A_{t-1} - r_{Bt-1}\overline{B} - O - A_t + \Delta H_t \right\} \right]. \tag{10}$$

The first order conditions are as follows:[6]

$$\partial V / \partial \Delta H_t = \beta^t (\Delta H_t + \lambda_t) = 0 , \tag{11a}$$

$$\partial V / \partial A_t = -\beta^t \lambda_t + \beta^{t+1} \lambda_{t+1} (1 + r_{At}) = 0 , \tag{11b}$$

$$\partial V / \partial \lambda_t = \beta^t \left[(1 + r_{At-1})A_{t-1} - r_{Bt-1}\overline{B} - O - A_t + \Delta H_t \right] = 0 . \tag{11c}$$

From equations (11a) and (11b),

$$\beta (1 + r_{At}) \Delta H_{t+1} = \Delta H_t . \tag{12}$$

The following transversality condition must be satisfied:

$$\lim_{T \to \infty} \beta^T \lambda_T A_T = 0 . \tag{13}$$

Equation (13) reduces to,

$$\lim_{T \to \infty} \frac{A_T}{\prod_{t=1}^{T} (1 + r_{At-1})} = 0 . \tag{13'}$$

It implies that A_T should not grow faster than its interest revenue.

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[6] The second order condition is always satisfied as follows:

$$d^2 L / dA_t^2 = \beta^t + \beta^{t+1} (1 + r_{At})^2 > 0 .$$

4. Transversality Condition and Credibility

4.1. The Model with Changes in Liabilities

We examine an optimal monetary policy and transversality condition in our model to derive some implications on central bank credibility. The optimal monetary policy must satisfy equation (7). It states only an intertemporal relation of ΔH, not each level of ΔH. Let us suppose $\Delta H_t=0$ for $t=1, \ldots, \infty$. This time path of ΔH satisfies equation (7) and produces the least value of the loss function (1). Then, the question is whether or not this monetary policy satisfies the transversality condition (8').

With the balance sheet constraint (4), reducing ΔH_t increases B_t, which reduces the profit and puts an expansionary pressure on future ΔH_{t+1} and B_{t+1}. The transversality condition (8') requires B_T expansion to be less than its interest payment. When B is positive, the transversality condition (8') implies,

$$\frac{B_T}{\prod_{t=1}^{T}(1+r_{Bt-1})}-\frac{B_{T-1}}{\prod_{t=1}^{T-1}(1+r_{Bt-1})}=\frac{1}{\prod_{t=1}^{T}(1+r_{Bt-1})}\left[B_T-(1+r_{BT-1})B_{T-1}\right]$$
$$=\frac{1}{\prod_{t=1}^{T}(1+r_{Bt-1})}\left[-\Delta H_T-r_{AT-1}\overline{A}+O\right]<0.$$

Then,

$$\Delta H_T>-\left(r_{AT-1}\overline{A}-O\right). \qquad (8'')$$

The monetary policy, $\Delta H_t=0$ for $t=1, \ldots, \infty$, satisfies the transversality condition if $r_{AT-1}\overline{A}-O>0$. In this case, the central bank can stop the monetary base expansion immediately. On the contrary, if $r_{AT-1}\overline{A}-O\leq 0$, then ΔH_T cannot be equal to zero in order to satisfy the transversality condition. ΔH_t must follow the intertemporal relation given by equation (7), and so ΔH_t at any t is positive since $\beta(1+r_{Bt})>0$.[7]

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[7] Even when $r_{AT-1}\overline{A}-O\leq 0$, the transversality condition holds at $\Delta H_T=0$, since $\lambda_T=0$ from equation (6a). Such a case seems to be unrealistic since it

The transversality condition (8') states that the central bank cannot increase B_T unlimitedly in order to control the monetary base. If it increases B_T beyond the transversality condition, it makes less profits or more losses and faces the pressure to supply more monetary base to finance the losses. To avoid the monetary base expansion, it must absorb the added monetary base by increasing B_T, which causes more losses. The liabilities become out of control, and the central bank is no longer sustainable. If it reduces or stops piling up liabilities, then it is forced to accelerate the monetary base growth. The monetary policy is not optimal, and we will have inflation. Either case damages the economy due to the central bank problem. Thus, the central bank loses credibility if the transversality condition is not satisfied.

4.2. The Model with Changes in Assets

In the model with changes in assets, the transversality condition is (13'), stating that A_T should not grow faster than its interest revenue. If this condition is not satisfied, the policy is not optimal, but it does not seem to cause any serious problem. The central bank can slow down the asset accumulation anytime by giving some part of profit to the government.

With similar calculation in Subsection 4.1, when A is positive, equation (13') is satisfied if the following holds:

$$\Delta H_T < r_{BT-1}\overline{B} + O. \tag{13''}$$

Unlike equation (8"), equation (13") always holds when ΔH_T=0. The asset holdings do not grow faster than its interest so long as the central bank conducts the optimal monetary policy.

The difference in transversality conditions in the above two models

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means that accumulating liabilities do not matter, and it might be because our model, especially the loss function, is too much simplified.

clarifies the difference between using the funds-supplying strategy or using the funds-absorbing strategy discussed in Subsection 2.3. The BOJ succeeded in exiting the quantitative easing by using the funds-supplying strategy, and such exit strategy corresponds to our model with changes in assets. Equation (13") holds, and the transversality condition is always satisfied. On the other hand, if such an exit strategy is not available, the bank needs to use the funds-absorbing strategy, which corresponds to our model with changes in liabilities. Equation (8") does not always hold, and when it does not hold, the bank cannot conduct appropriate monetary policy and loses credibility as discussed above. Our discussion warns that central banks should be cautious in using the funds-absorbing exit strategy.

4.3. Capital, Balance Sheet, and Interest Rates

Our model shows that a central bank might lose credibility when it needs to take the strategy to expand the interest bearing liabilities. It is noteworthy that the condition stated as equation (8") does not depend on the capital. The central bank can be credible even if its capital is small or negative. Figure 2 exhibits a numerical example of such an extreme case. Since $r_{AT-1}\overline{A} - O \geq 0$, the central bank sets ΔH_t=0 for t=1, ..., ∞. The capital becomes negative and decreasing, but the value of transversality condition function (8') is converging to zero. The capital becomes negative, but the central bank can be credible because it can conduct the optimal monetary policy.

Insert Figure 2 around here.

However, the capital does affect the situation of central bank. Less capital K_0 implies less asset holdings $\overline{A}$, which makes the condition (8") more restrictive. A central bank may suffer a large loss due to foreign exchange loss or the cost to deal with domestic financial crisis, as the troubled central banks discussed in Subsection 2.1. Such a loss decreases the central bank's asset holdings and capital. In our model, they correspond to small K_0 and $\overline{A}$. Small $\overline{A}$ incurs sustained loss and makes it difficult to control the monetary base by changing liabilities as shown by equation (8"). Thus, not the capital alone but the balance sheet as a whole is important for the bank credibility.

It should also be noted that equation (8") does not include r_B. Some experiences of troubled central banks discussed in Subsection 2.1 indicate that a rise in the interest rate on central bank liabilities deteriorated the situation, but our model argues that it does not depend on such an interest rate change whether or not the central bank is sustainable.

A rise in the liability interest rate, however, affects the time path of ΔH. Equation (7) shows that the relative sizes of ΔH at t and t+1 depend on $\beta(1+r_{Bt})$; higher r_{Bt} leads to higher ΔH_t to suppress the expensive B_t expansion. If the interest rate r_{Bt} is smaller than the discount rate implied by β, then $\beta(1+r_{Bt})<1$. ΔH_t is smaller than ΔH_{t+1}, which keeps the immediate monetary base expansion and inflation mild. ΔH_{t+1} is larger, however, and so the monetary base expansion and inflation gradually accelerate. If r_{Bt} is larger than the discount rate, then $\beta(1+r_{Bt})>1$, and ΔH_{t+1} is smaller. Though ΔH gradually shrinks, ΔH has a positive lower bound in the future as shown by the transversality condition (8"), and so ΔH in early periods must be large. Thus, the higher liability interest rate implies an immediate large monetary base expansion and high inflation. Furthermore, if the interest rate remains high, ΔH will approach to zero, and the transversality condition will not be satisfied sooner or later.

The transversality condition depends on some of the initial and exogenous variables, and it should be noted that these variables are largely affected by what has happened before t=1. A loss from domestic or foreign asset transactions or a cost to deal with domestic financial crisis has an impact on $\overline{A}$ and/or O, or purchasing low-return assets keeps r_{At} low for a while. Given these variables, our model checks whether or not the central bank can reduce the monetary base expansion at t=1,.., ∞. Thus, one of the important implications from our simple model is that a central bank needs to be ready for a large economic shock so that it does not lose its balance sheet soundness and profitability due to the shock.[8]

5. Conclusion

In this paper, we have examined how a central bank can keep its credibility when exiting monetary easing. For this purpose, we have developed a dynamic optimization model of a central bank and examined the central bank credibility. Unlike the existing literature, we have incorporated the central bank's balance sheet and profit into the model and considered the central bank behavior that minimizes the loss function on infinite time horizon by controlling the monetary base. We also have taken into account the difference between a funds-supplying exit strategy and a funds-absorbing exit strategy. With each exit strategy, the model has analyzed whether or not the central bank can stop the monetary base expansion, and our analysis has found the followings.

First, the central bank credibility is closely related to the transversality condition. If the condition is not satisfied, the central bank is not sustainable or it is forced to conduct inappropriate inflationary policy. Thus, without the condition satisfied, the central bank loses credibility.

[8] Stella [2003] emphasizes the importance to withstand economic shocks and argues that central banks need "financial strength."

Second, the transversality condition differs depending on using either the funds-supplying strategy or funds-absorbing strategy, and the condition for using the funds-absorbing strategy is not always satisfied. Central banks may face a sudden necessity for monetary tightening, and the funds-supplying strategy may not be available. Our analysis, however, warns that central banks should be cautious in using funds-absorbing measures for tightening as the transversality condition is not always satisfied.

Finally, the transversality condition and credibility is not directly related with the central bank capital. They depend on the initial conditions and exogenous variables, which constitute the balance sheet soundness. The central bank's balance sheet should be sound enough to generate no sustained loss, even when the central bank suffers a large economic shock.

Our model is a simple one, and many extensions are possible, such as introducing uncertainty, profit transfer to the government, and so on. They remain for the future study.

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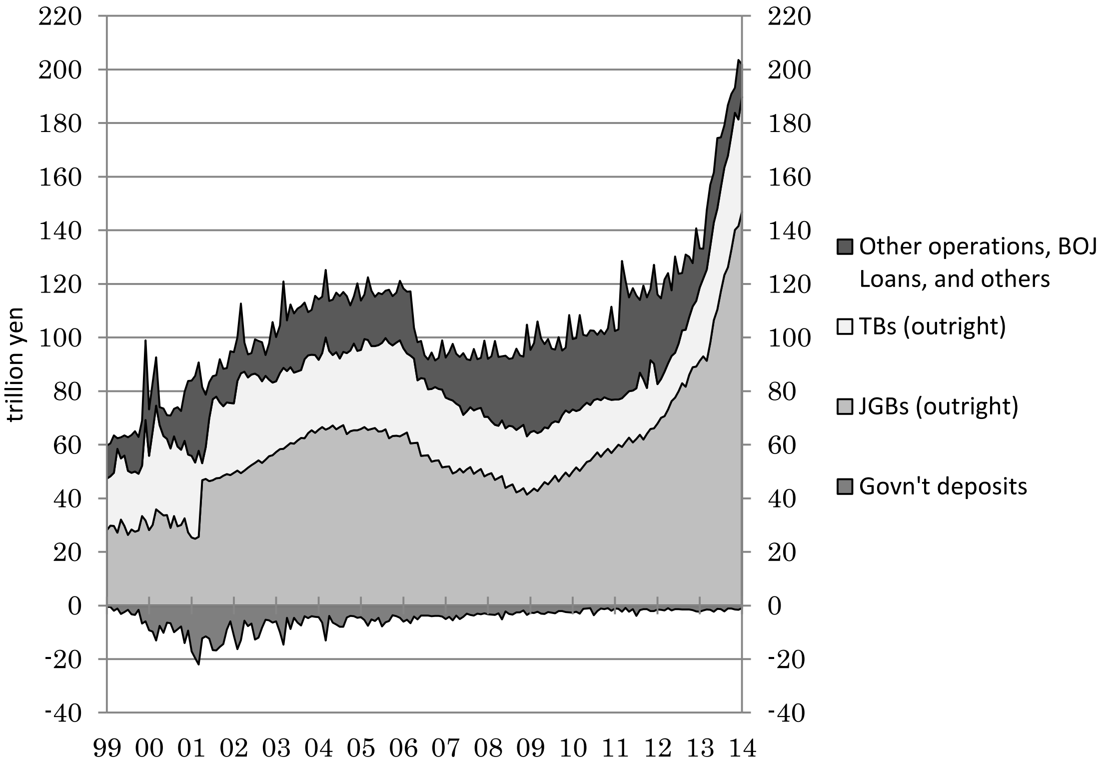
Figure 1. Monetary Base in Japan



Source: BOJ.

Table 1. Balance Sheet of the Bank of Japan

Domestic Assets	226.4	Monetary Base	196.3
(JGBs	147.0)	Other Liabilities	29.8
(Risky Assets	9.6)	Capital	6.1
Foreign Assets	5.8		

Notes: Trillion yen in January 2014.

Monetary base does not include coins. Capital includes appropriate reserve funds. Risky assets are CPs, corporate bonds, stocks, ETFs, and REITs. Foreign assets include gold.

Source: BOJ.

Table 2. The Bank of Japan's Exit Strategy

		Mar. 06-Aug. 06	Mar. 06-Aug, 08
Monetary Base		-23.69	-22.98
JGBs		-8.55	-19.35
	Purchases	7.13	36.16
	Redemption	-15.67	-55.50
TBs		-4.88	-7.84
Funds-Supplying Operations against Pooled Collateral, RAs, and Bills		-24.64	-14.39
	Funds-Supplying	-25.44	-15.20
	Funds-Absorbing	0.80	0.80
BOJ Loans and Others		14.38	18.60

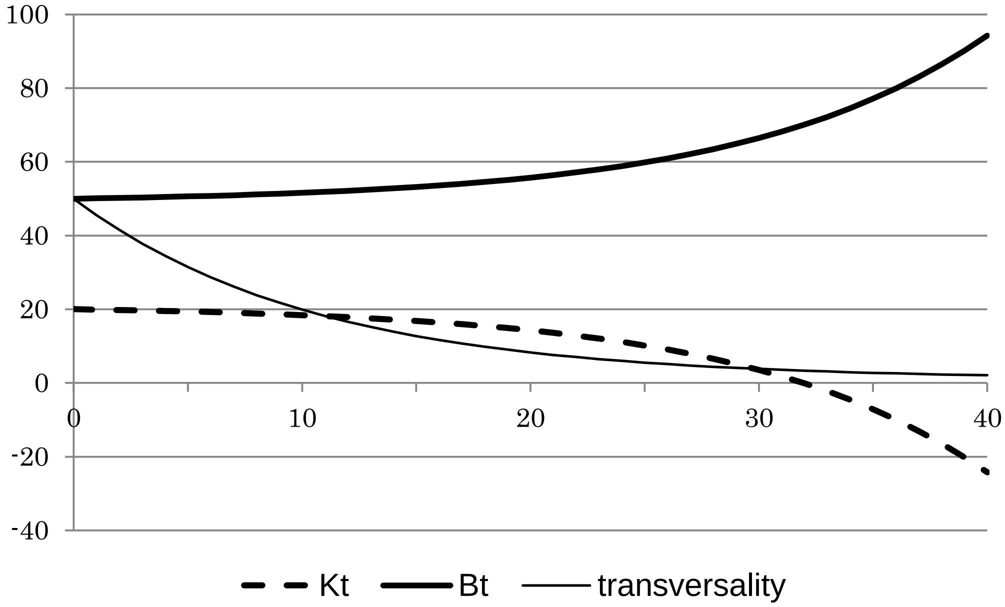
Notes: Flow amounts. Trillion yen.

Source: BOJ.

Table 3. Central Bank Balance Sheet

Assets (A_t)	Monetary Base (H_t) Interest Bearing Liabilities (B_t) Capital (K_t)

Figure 2. A Numerical Example



Notes: The following is assumed.

H_0=30, K_0=20, B_0=50, O=5.1, $r_{At} = r_{Bt}$ =0.1, β＝0.05.

“Transversality” is the value of $B_T \Big/ \prod_{t=1}^{T} (1 + r_{Bt-1})$.

ΔH_t=0 for t=1,.., ∞ satisfies equation (8”) as $-(r_{At}\overline{A} - O) = -4.9$.